PRESENTATION OF MATHEMATICAL ECONOMICS SYLLABUS FOR ECONOMICS HONOURS UNDER CBCS,

UNIVERSITY OF CALCUTTA

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Introductory Remarks

- The paper/Course is actually **'Mathematical Methods in Economics'** *and not* 'Mathematical Economics'.

- There are two parts of the Course. The first is 'Mathematical Methods in Economics-I' for Semester-I (100 marks, 5+1=6 credits) and the second part is Mathematical Methods in Economics-II' for Semester-II (100 marks, 5+1=6 credits)

- The idea behind the Course is to give the students some background of Mathematical tools that can be used for analyzing economic ideas.

- A student may opt for Mathematics as Generic Elective in the 3^{rd} and 4^{th} Semesters instead of 1^{st} and 2^{nd} semesters. It may happen that the 3^{rd} and the 4^{th} Semester Mathematics Generic Elective Courses have fewer applications in the field of Economics.

- So the Course Instructor who will teach Mathematical Methods in Economics should first give emphasis on the Mathematical Methods and then can use Economic examples.

- For this purpose Chiang and Wainwright's book has been considered as the Text for both 1^{st} and 2^{nd} semesters.

- Useful references are Sydaster and Hammond and also Simon and Blume.

- Gibbon's book is essential for the Game Theory part.

- An Error: Please ignore the reference Hogg and Craig.

- Mathematical Methods in Economics should be linked with Micro and Macro Courses as well.

- Let us first of all have a look at the syllabus for Mathematical Methods in Economics-I with illustrations:

Course Content with Illustrations

Economics Core Course II: ECO-A-CC-1-2-TH-TU

Mathematical Methods in Economics-I

Total Marks: 100 [Theory(Th) 65 + Tutorial(Tu) 15 +

Internal Assessment 10+Attendance: 10]

Total Credits: [5(Th)+1(Tu)]=6 ,

No. of Lecture hours: 75, No. of Tutorial contact hours: 15

[For Semester-I]

ECO-A-CC-1-2-TH

1. Preliminaries 10 lecture hours

- Sets and set operations; functions and their properties; number systems.

- Convex sets; geometric properties of functions: convex functions, their characterizations, properties and applications; further geometric properties of functions: quasi-convex functions, quasi-concave functions, their characterizations, properties and applications.
- Limit and continuity-Different Limit Theorems with proof-concept of first principle.
- Uses of the concept of continuity.

  [- For this part out of 10 lectures ***at least 5 lectures*** should be confined to convex sets, convex functions, quasi-convex and quasi-concave functions due to its wide applicability.

  - It will be easier for the students to have an idea about the nature of the utility function if they have knowledge about quasi-concave function]

2. Functions of one real variable 10 lecture hours

- Continuous functions of different types and their graphs-quadratic, polynomial, power, exponential, and logarithmic.
- Concept of derivatives. Limits and derivatives. L' Hopital's rule .Graphical meaning of derivatives. Derivatives of first

and second order and their properties; convex, concave and linear function.

- Application in economics- concept of marginal. Concept of elasticity. Concept of average function

[-This part of the Course is consistent with the Microeconomics Course. Various types of demand functions can be introduced to the students and they may be asked to find out the price elasticity.

- Students should know how to link price elasticity with total expenditure/total revenue. They should also know the relationship among price elasticity, average revenue and marginal revenue.

-Students will get basic idea on this from Microeconomics paper and in this paper they should know ***how to solve tricky problems*** related to elasticity of demand.

- Care is to be taken for L 'Hopital's rule and on the basis of a CES production function one can demonstrate that how Cobb-Douglas production function can be considered as a special case of CES production function using L' Hopital's rule.]

3. Single variable optimization 10 lecture hours

- Local and global optima; Geometric characterizations; characterizations using calculus. Significance of first and second order conditions.
- Interpretation of necessary and sufficient conditions with examples.
- Applications in Economics- profit maximization and cost minimization.

[- Out of 10 lectures 5 lectures should be allotted to local and global optima due to its wide applications in Economics.

- Students should have a good idea regarding the implications of necessary and sufficient conditions. ***They have the wrong notion that necessary condition means first order condition and sufficient condition means second order condition.***

The concept can be illustrated with the help of examples. I am providing here one such example.

Example:

$$Y = X_1^2 + X_2^2 + (X_1 - X_2)$$

Given $X_1 > 0,\ X_2 > 0$ we have $Y > 0$ if $X_1 > X_2$

Thus $X_1 > X_2$ is sufficient for $Y > 0$

Suppose we have $Y = (X_1 - X_2)$

Here $Y > K > 0$ only if $X_1 > X_2$

Thus $X_1 > X_2$ is necessary for $Y > K > 0$

Finally $Y > 0$ iff $X_1 > X_2$

Thus $X_1 > X_2$ is necessary and sufficient for $Y > 0$

- Basic concepts of profit maximization and cost minimization will help the student to have a basic idea about production, cost and markets. This part of the course is thus fully consistent with the Microeconomics Course of 1st semester]

4. Integration of functions 15 lecture hours

- Integration of different types of functions;
- Methods of Substitution and integration by parts.
- Applications in economics- obtaining total from the marginal.

[- Idea about integral calculus is not only necessary to obtain total from the marginal but also it helps a student to have an idea about the concept of present value.]

5. Difference Equations 15 lecture hours

- Finite difference; Equations of first and 2nd orders and their solutions
- Application in Economics- Cobweb model, Multiplier-Accelerator model.

[- An idea about solving a first order and second order difference equation through particular solution and trial solution will help the students to tackle the Cobweb Model.

- The idea of introducing multiplier-accelerator here may look awkward. However, it is to be noted that multiplier-accelerator interactions is one of the most important applications of difference equation.
- The teacher will have to give some basic idea of Simple Keynesian Multiplier first (1 class). Then the teacher can introduce lagged consumption function in an otherwise Simple Keynesian Model and can examine the consequence (1 class). Next the teacher can introduce the concept of accelerator and

can focus on Multiplier-Accelerator Interaction (2 classes). So out of 15 at least 4 classes are to be allotted for multiplier-accelerator interaction and related issues]

6. Game Theory 15 lecture hours

- Concept of a game, strategies and payoffs
- Zero-sum games- maxmin and minmax solutions
- Dominant Strategy Equilibrium
- Nash equilibrium
- Nash equilibrium in the context of some common games – Prisoners' Dilemma, Battle of Sexes, Matching Pennies

[- The first two bullets can be covered from Chiang or any one of the remaining suggested references.

- From Dominant Strategy till the end of proposed Game Theory topics are to be covered from Gibbons.

- Only static games under complete information will be covered.

- Nash Equilibrium is to be supplemented by Cournot-equilibrium.

- Prisoner's Dilemma, Battle of Sexes and Matching Problems are to be illustrated clearly in the class]

ECO-A-CC-1-2-TU

Tutorial contact hours: 15

[For tutorial examination five topics can be considered for demonstration lecture like (this is not final and is considered here just for illustrative purpose):

- Sets and functions and their applications in Economics
- Concept of elasticity , marginal and the average functions
- Local and global optima and necessary and sufficient conditions : applications in Economics
- Applications of Difference Equations in Economic Models
- Dominant Strategy and Nash Equilibrium in Static Games with Complete Information

Students may be asked to deliver demonstration lecture on any one of the five topics (10 marks) **in front of an external examiner** followed by a viva-voce examination **to be taken by the external examiner** on the basis of the demonstration lecture (05 marks)]

Texts :

Alpha C. Chiang and Kavin Wainwright : Fundamental Methods of Mathematical Economics, Mc Graw Hill, 2005.

Gibbons R. Game Theory for Applied Economists

References

K. Sydsaeter and P. Hammond, Mathematics for Economic Analysis, Pearson Educational Asia: Delhi, 2002.

Mukherji and S. Guha: Mathematical Methods and Economic Theory, Oxford University Press, 2011.

Hands, D. W.: Introductory Mathematical Economics, Second Edition,2004

Silberberg ,E. and Suen, W.: The Structure of Economics : A Mathematical Analysis, Third edition, Mc-Graw Hill, 2001

Apostol T.M. : Calculus, Volume 1, One-variable calculus, with an introduction to linear algebra, (1967) Wiley, ISBN 0-536-00005-0, ISBN 978-0-471-00005-1.

K. G. Binmore, Mathematical analysis, Cambridge University Press, 1991.

R.V. Hogg and A.T. Craig , An Introduction to Mathematical Statistics, Third Edition, Amerind, New York, London.

We now pass on to the Course Content with Comments for the Mathematical Methods in Economics-II syllabus

Economics Core Course IV: ECO-A-CC-2-4-TH-TU

Mathematical Methods in Economics-II

Total Marks: 100 [Theory(Th) 65 + Tutorial(Tu) 15 + Internal Assessment 10+Attendance: 10]

Total Credits: [5(Th)+1(Tu)]=6 ,

No. of Lecture hours: 75, No. of Tutorial contact hours:15

[For Semester-II]

ECO-A-CC-2-4-TH

1. Matrix Algebra 10 lecture hours

- Matrix: its elementary operations; different types of matrix.
- Rank of a matrix.
- Determinants and inverse of a square matrix.
- Solution of system of linear equations; Eigen values and Eigen vectors.
- System of nonlinear equations- Jacobian determinant and existence of solution.
- The concept of comparative statics

[-Students should have good idea of matrix, determinant, inverse of a matrix, eigen values , eigen vectors etc. Out of total 10 lectures a teacher should deliver ***at least 6 lectures*** to develop these concepts.

- The concept of non-linear equations and Jacobian along with the concept of comparative statics can be taught in the context of stability under differential equations.

-An idea about non-linear equations will give some idea about the use of Taylor's series in Economics

-Sometimes the students cannot answer the question like "How one can use Cramer's rule (which is a part of linear algebra) in examining the impact of government expenditure in IS-LM model when the equations of IS-LM model are non-linear?"

- Here IS-LM model is just an example. My point is that answer to these types of questions will help the students]

2. Function of several variables 15 lecture hours

- Continuous and differentiable functions: partial derivatives and Hessian matrix. Homogeneous and homothetic functions.

- Euler's theorem, implicit function theorem (without proof) and its application to comparative statics problems.
- Economic applications- the idea of level curves, theories of consumer behaviour and theory of production.

[- Students should have a clear idea about different types of continuous and differentiable functions. Let us consider two examples:

Recently in an interview I asked a candidate draw the total product, average product and the marginal product curve for the production function

$$Q = \min(\frac{L}{a}, \frac{\overline{K}}{b})$$

The candidate could not answer it. In most cases students wrongly answer it as a L-shaped isoquant though it is specifically mentioned that $\overline{K}$ is given and one will have to find out total product, average product and marginal product.

Another example is draw the isoquant in case of the production function

$$Q = \min(2L + K, 2K + L)$$

The students in most cases wrongly answer it as a L-shaped isoquant though the actual is that it will be a negatively sloped straight line isoquant with a kink.

- Idea of Euler's theorem will help the students to understand the theorems related to Consumer Behaviour and the Theory of Distribution.
- Functions of several variables and partial derivatives will also help the students to have an idea of the mathematical interpretation of consumer behavior introduced in 1st semester.]

3. Multi-variable optimization 35 lecture hours

- Optimization of nonlinear functions: Convex, concave, and quasi-concave functions; Unconstrained optimization.
- Constrained optimization with equality constraints- Lagrangian multiplier method; role of Hessian determinant.
- Inequality constraints and Kuhn-Tucker Conditions.
- Value function and Envelope theorem; Economic applications – consumer behaviour and theory of production.
- Optimization of linear function: Linear programming; concept of slack and surplus variables (graphical solution

only). Concept of convex set. The Duality Theorem

- Economic Applications of Linear programming

[-This is one of the most important topics in Mathematical Economics due to which 35 lectures are allotted

-Idea of constrained optimization will help the students to have proper idea of the theory of consumer behavior. Special emphasis should be given on interpretation of Lagrange multiplier, positive monotonic transformation of the utility function, demand functions and the Slutsky equation, difference between cost equation and a cost function, derivation of the supply function from the production function etc.

- the idea of Bordered Hessian will help the students to know about the second order conditions in case of constrained optimization

-Idea of envelope theorem is introduced for the first time in Honours. It will help the students to have an idea of maximum and minimum value functions.

-As envelope theorem is introduced it is expected that students should have some idea about Shephard's Lemma and Roy's identity

- The teacher can shuffle the syllabus and can teach linear programming first and then Kuhn-Tucker conditions under non-linear programming]

4. Differential Equations 15 lecture hours

- Solution of Differential equations of first order and second order of linear differential equations.
- Economic application-price dynamics in a single market-multimarket supply demand model with two independent markets.
- Qualitative graphic solution to 2x2 linear simultaneous non-linear differential equation system- phase diagram, fixed point and stability. Economic applications in microeconomics and macroeconomics

[- The stability analysis is essential for the students to understand that comparative static exercises cannot be analyzed unless we have a stable equilibrium. The role of simultaneous non-linear differential equation is important in this context (see Chaing for an excellent treatment). For illustrative purpose let us consider the IS-LM example. **[Stability analysis of IS-LM model can be considered here as an example but the details of IS-LM model will be discussed in semester III].**

We consider

$$\dot{y} = f(EDG) = f[C(y) + I(r) - y]$$

where $f' > 0$ and $f(0) = 0$.

Thus the IS curve can be interpreted as the locus of y and r such that $\dot{y} = 0$ i.e. the product market is in equilibrium.

$$\dot{r} = g(EDM) = g[ky + L_2(r) - M]$$

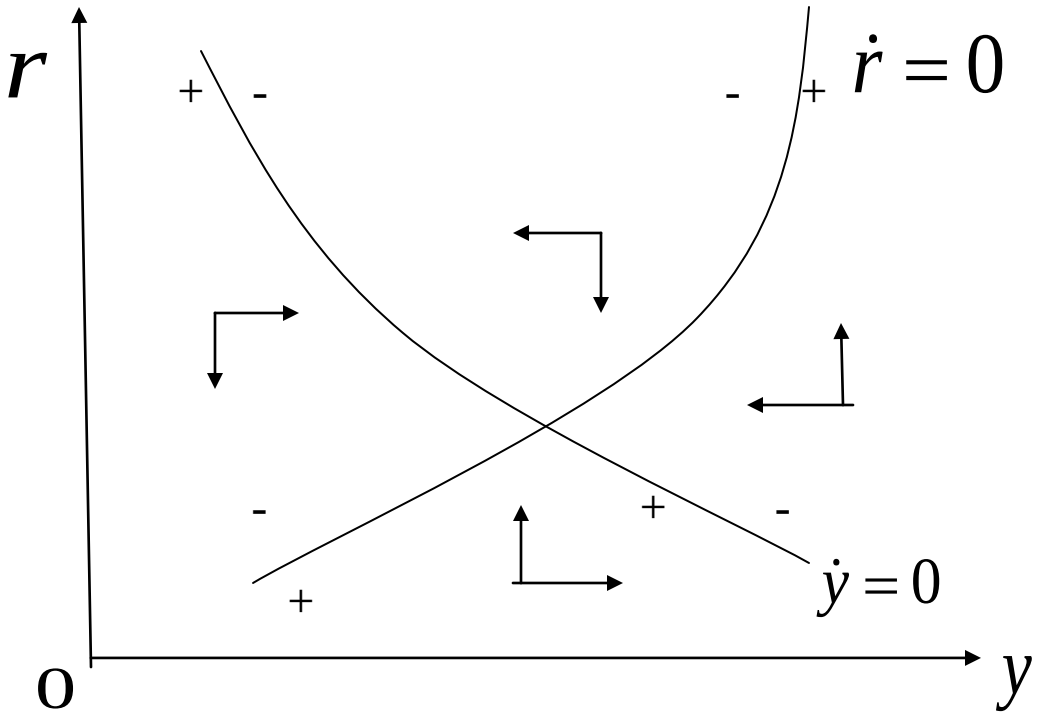
where $g' > 0$ and $g(0) = 0$. Given $P = 1$.

Thus the LM curve can be interpreted as the locus of y and r such that $\dot{r} = 0$ i.e. the money market is in equilibrium.

$$\frac{\partial \dot{y}}{\partial y} = f'[C' - 1] < 0$$

and

$$\frac{\partial \dot{r}}{\partial r} = g'[L_2'] < 0$$



ECO-A-CC-2-4-TU

Tutorial Contact hours: 15

[For tutorial examination five topics can be considered for demonstration lecture like (this is not final and is considered here just for illustrative purpose):

- Economic Interpretation of LPP and the Duality Theorem
- Non-linear Programming Problem and the Kuhn –Tucker conditions : Economic Interpretations
- Constrained Optimization in Economics

- Use of matrix algebra in explaining Comparative Statics in Economic Theory

  Non-linear Differential Equations and Phase Diagram Techniques with Economic Applications

Students may be asked to deliver demonstration lecture on any one of the five topics (10 marks) **in front of an external examiner** followed by a viva-voce examination **to be taken by the external examiner** on the basis of the demonstration lecture (05 marks)]

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Alpha C. Chiang and Kavin Wainwright : Fundamental Methods of Mathematical Economics, Mc Graw Hill, 2005.

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Carl Simon and Lawrence Blume. Mathematics for Economists, W. W. Norton and Company, 1994

A. Mukherji and S. Guha: Mathematical Methods and Economic Theory, Oxford University Press, 2011.

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Silberberg ,E. and Suen, W.: The Structure of Economics : A Mathematical Analysis, Third edition, Mc-Graw Hill, 2001

K. G. Binmore, Mathematical analysis, Cambridge University Press, 1991.